Optimal Filtering: project assignment

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In this project assignment you will work with the design of a motion tracking filter for a global positioning system (GPS) receiver. The GPS receiver is going to be used to track the motion of car driving around a race track. The attached MATLAB file `GPSdata.mat` *include the true (position) trajectory of the car and the pseudo-range measurements recorded by the GPS receiver. Your task is to design and tune a Kalman filter that tracks the motion of the car.*

1 Problem Description

Today the GPS consists of 32 satellites that orbit the earth at an altitude of 20 000 km. These satellites constantly transmits time encoded signals that may be received by a GPS receiver at earth. The GPS receiver uses the received signals to calculate the distances between the GPS receiver and the satellites. The distance estimates are generally referred to as pseudo-range measurements since they also include a range offset caused by the offset in the GPS receiver clock. The pseudo-range measurement to the i^{th} satellite at time k can be modeled as

$$y_k^{(i)} = \|\mathbf{p}_k^{(i)} - \mathbf{p}_k^{(\text{rec})}\| + c \cdot \Delta t_k + v_k^{(i)}, \tag{1}$$

where $\mathbf{p}_k^{(i)}$ and $\mathbf{p}_k^{(\text{rec})}$ is the position of the satellite[1] and the receiver, respectively. Further, c is the speed of light and $v_k^{(i)}$ is the measurement noise, which can be assumed zero-mean and have the variance $\mathsf{E}\big[v_k^{(i)} v_l^{(j)}\big] = \sigma_r^2\, \delta_{i-j,k-l}$. Moreover, the clock offset Δt_{k+1} in the receiver can be modeled as

$$\begin{pmatrix} \Delta t_{k+1} \\ \dot{\Delta} t_{k+1} \end{pmatrix} = \begin{pmatrix} 1 & T_s \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \Delta t_k \\ \dot{\Delta} t_k, \end{pmatrix} + \mathbf{w}_k^{\text{clk}}, \tag{2}$$

where T_s is the sampling period and $\mathbf{w}_k^{\text{clk}}$ is white noise with the covariance

$$\mathbf{Q}_k^{\text{clk}} = \begin{pmatrix} S_\phi T_s + \frac{1}{3} T_s^3 S_f & T_s^2 S_f \\ T_s^2 S_f & S_f T_s \end{pmatrix}. \tag{3}$$

Here S_ϕ and S_f are the power spectral density of the phase and frequency process noise (both of these noise processes are assumed to be white).

*These exercises are heavily inspired by exercises used by Isaac Skog (LiU) and Mats Bengtsson (KTH).

[1] The position of the satellite $\mathbf{p}_k^{(i)}$ is known by the receiver, but the satellites from which the GPS receiver can receive signals varies with time and the position of the receiver.

If the signals from four or more satellites are received then it is possible to, without introducing a model for the motion of the GPS receiver or the clock-offset, calculate a position estimate using a nonlinear least squares algorithm. The attached MATLAB function `NonLinearLeastSquares.m` is an implementation of such an algorithm. The function also gives an illustration of how to handle the data content in the `GPSdata.mat` file.

2 Tasks

Approach the problem by performing the following tasks:

Task 1 Plot the trajectory of the car and the position estimates calculated by the GPS receiver using the nonlinear least squares algorithm.

Task 2 Reference [1] provides a survey of motion dynamic models for target tracking. Combine the constant velocity or constant acceleration model described on pages 1333–1337 in the paper with the clock model in (2) and implement a Kalman filter that tracks the motion of the car.

- **a)** Specify the state-space and observation models that you use in your filter.
- **b)** Is the stochastic process described by the state-space equations a stationary process? Does it matter?
- **c)** What happens if the GPS receiver stops receiving signals from the satellites? How will that affect the state covariance?

Task 3 Plot the position estimation errors (each coordinate axis in a separate plot) and clock offset and clock drift estimation errors. Include the 3σ confidence intervals based on the state covariance matrix output by the filters. Further, in the plots with the position estimation errors, also include the results of the nonlinear least squares algorithm.

- **a)** What happens when the covariance of the process noise of the motion dynamic model becomes very small?
- **b)** What happens when the covariance of the process noise of the motion dynamic model becomes very large?
- **c)** Discuss how the innovation and the 3σ confidence intervals can be used to tune the filter?
- **d)** Throughout the course we have studied linear estimators (filters) that are optimal in the mean square error sense. Name at least two reasons why the implemented filter may not be optimal?

3 Reporting

Please write a technical report (not a thesis!) illustrating your examples and document your conclusions. The report and numerical results should be individually prepared by each student. Write the report in the format of a conference paper, for example using the style file and templates available at

`https://www.ieee.org/conferences/publishing/templates.html`.

Hand in your report and zip-file containing all your code.

4 Data Description

The provided data file `GPSdata.mat` contains the following variables.

`gps_data`	$1 \times M$ array of struct with the fields:
`.Satellite`	Name of satellite
`.Satellite_Position_NED`	Position of the satellite
`.PseudoRange`	Measured pseudo ranges
`ref_data`	Struct with the fields:
`.traj_ned`	True trajectory of the vehicle
`.x_clk`	True clock state (offset, drift)
`.Ts`	Sample period
`.PSD_clk`	Power spectral density of clock process noise
`.s2r`	variance of range measurement error
`.c`	speed of light

References

[1] X. R. Li and V. P. Jilkov, "Survey of maneuvering target tracking. Part I. Dynamic models," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 39, no. 4, pp. 1333–1364, Oct. 2023. [Online]. Available: https://doi.org/10.1109/TAES.2003.1261132